

Choosing the Discount Rate: A Fairy Tale

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Abstract:

A tale involving a Prince, his wizard, a bargaining magical frog, the wise old rabbit, and the Prince's neighboring rulers illustrates a number of known and less well known hazards in choosing the discount rate for evaluating investments. The tale reveals a new twist to the determination of the actual value of the opportunity cost of capital that involves deviations in estimated project value caused by the professional inability of forecasters, evaluators and decision makers.

The Proposition

It was another beautiful day in the Enchanted Kingdom and the young Prince was taking his daily walk past the pond. A large green frog sitting on a lily pad rolled his eyes at the Prince and said "Hi, Prince."

"Hi, frog." the Prince replied without hesitation or surprise, for talking animals were the norm in the Enchanted Kingdom. "What's happening?" the Prince inquired.

"You are doing just a great job of making this Kingdom work, Prince." the frog replied. "I want to reward you. So, I am going to give you a choice of two alternatives and you can take your pick of whichever one you want." continued the frog. "Here's the deal."

"I will give you the dam on the river that you wanted to build." continued the frog. "The dam will supply energy to process your crops and make goods to sell in other neighboring kingdoms continuously for the next fifty or more years. It will also provide flood control and recreational activities."

"Wow!" exclaimed the Prince. "What is the other alternative?"

"Alternatively, I will give you a gold mine." explained the frog. "Unknown to you, there is a rich deposit of gold and pyrite buried deep in the cliffs next to the river. Mining this rich deposit will provide much wealth to you over the next ten to twenty years. You can then use this wealth to help your subjects in any way you desire."

"However," continued the frog, "there are certain drawbacks to each offer. The dam will also flood valuable land and disrupt aquatic life. The gold mine will require reclamation when the mine closes. It will also produce acid water that will need treatment on a declining cost basis for a thirty-year period. "

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"The money it takes to build the dam or construct the gold mine is about equal. While it appears the benefits strongly outnumber the costs in both alternatives, you may conclude otherwise. Both projects should provide a means to a higher standard of living for your subjects. I know you have wanted to improve living conditions for some time, but whenever there was spare income, there were always other things that seem to be more important. Although this is an Enchanted Kingdom, money still does not grow on trees, you know." joked the frog.

The frog shifted on the pad a bit as he eyed a nearby buzzing fly and replied, "So, the choice is yours."

"Oh, oh." said the Prince to himself as much to the frog. "This is not going to be easy. Frog, can I give you my answer at this time tomorrow? I need to consult my Wizard about this."

"Sure." replied the frog as his attention clearly shifted toward the nearing fly. "See you tomorrow."

The Wizard's Valuation

The Prince ran back to the castle to summon the Wizard and his advisors. Once they were gathered in the Great Room, the Prince explained the frog's proposition. The Wizard gathered his staff and assigned tasks required for the completion of a risk adjusted discounted cash flow analyses complete with sensitivity and scenario analysis. The Royal Computer hummed all night as the Wizard considered population projections, future energy and agricultural product prices, forecast GKP (gross kingdom product), employment levels, and other necessary data. Finally at mid-morning the next day amid piles of computer output the Wizard declared that his analysis was complete.

"Prince, I have an answer for you." the Wizard stated wearily, but with apparent pride. "The Net Present Value of the gold mine is definitely higher than that of the dam. You should choose the gold mine. The mine will last only twenty years, but will provide significant revenue and jobs immediately. Although the benefits of the dam last for a full fifty-year period, the annual benefits are smaller than those obtained from the gold mine. In addition, the environmental costs of the mine are not paid until far into the future. Any reasonable discount rate makes the present value of the dam small compared to the present value of the gold mine."

"Great!" exclaimed the Prince. "But, tell me more about your assumptions and the choice of the discount rates. What rates were used and how were the cash flows calculated?"

To that, the Wizard replied with the following explanation:

"Figure 1 compares the constant dollar cash flows of the mine and dam. Both projects require the same amount of capital and last for the same number of years. The

mine has large positive cash flows in early years, a large reclamation cost in year 20, and declining negative cash flows in years 21 to 50. The dam has lower cash flows than the mine in the early years, but continues to have positive constant cash flows for the entire 50-year period. Because of the different patterns of benefits and costs, the choice of discount rate will be important in choosing between the two projects."

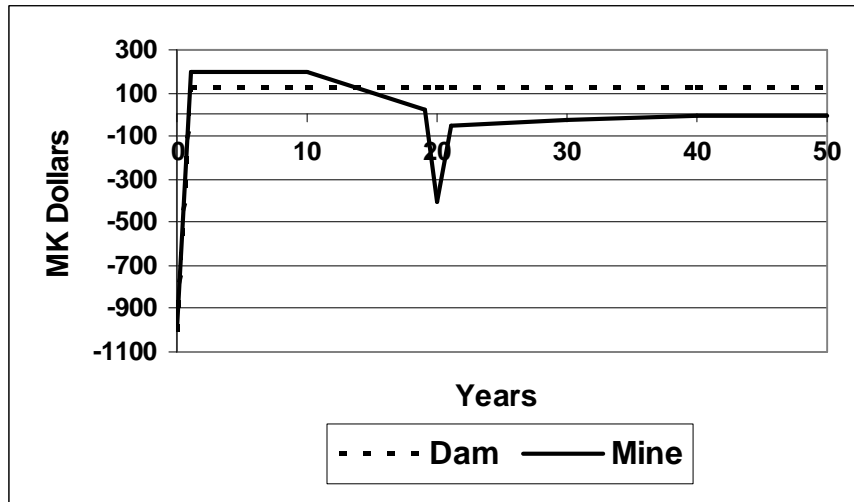


Figure 1. Cash Flow Profiles of the Dam and Mine (constant dollars)

"While we all agree that the opportunity cost of capital (OCC) is the proper discount rate, determining the value of OCC is difficult. There are numerous rates we can consider, as are shown in Table 1."

Table 1. Candidate Rates of Return (Current After Tax)

WACC (calculated)	13.97%
Risk free alternative	5.50%
Cost of debt	11.00%
Historical rate of return	14.00%
Risk adjusted rate of return	14.00% + risk
Hurdle rate	15.00%
Social rate	14.00% - social
Temporal rate (Intergenerational)	14% - temporal adjustment
Tax rate	0.00% (does not tax itself)
Inflation rate	2.50%

"Note that any rate in current dollars, such as the historical rate and the cost of debt, must be adjusted for inflation before using to discount the constant dollar cash flows of the two projects. Also, these rates must all treat taxes consistently. Of these rates, the weighted average cost of capital (WACC) is the one we believe most accurately portrays the OCC."

"We used a discount rate of 12.0%, which is close to the actual calculated value of 11.47% of the after tax, constant dollar WACC. Although, both projects have different risks, we believe the overall amount of risk for each project is the same."

"At any reasonable discount rate (above 9.0%) that approaches the WACC, the mine will have the higher NPV. This is shown in Figure 2. Therefore, we recommend the mine."

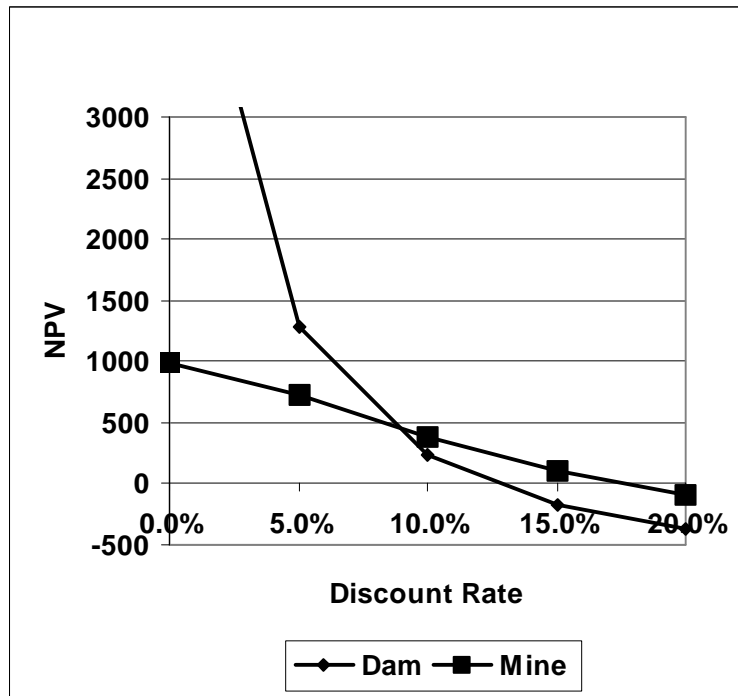


Figure 2. Sensitivity Analysis of NPV and Discount Rates of Mine and Dam

Pollution Abatement Sinking Fund at Safe Rate

The Price considered this explanation for several moments, and then replied "I still have a number of questions. First, I am concerned that the process of discounting at a higher discount rate disguises the problems that may be encountered with the pollution abatement in years 20 through 50 for the mine. I believe the cost of the future pollution abatement should be paid as a current expense to produce a sinking fund that will finance the pollution abatement that occurs at the end of the mine's useful life. Since we must invest these funds in a safe manner, the discount rate for these funds should be around 3.0% rather than the WACC. The WACC should be used to determine the present value of the cash flows after deducting for the sinking fund contributions. What does this do to NPV of the mine?"

"Oh!" replied the Wizard. "What you suggest is based on the same principle as the Hoskold formula, but instead of creating a sinking fund at a safe rate to finance new operations after depletion of the current mine, you want to finance a pollution abatement

program. This is easily done.” the Wizard continued as he pressed a few keys on his laptop.

“The new cash flow profile of the mine is now shown in Figure 3. The mine now has lower cash flows in years 1 through 20 but no negative cash flows due to pollution abatement from years 20 through 50. Figure 4 shows the NPV of the dam and the two versions of the mine. The modified mine cash flows are discounted at 12% and the sinking fund is accumulated at 3%. This means the NPV of the mine will decrease. The mine is still the preferred project at any discount rate over 10%, although by a smaller margin.”

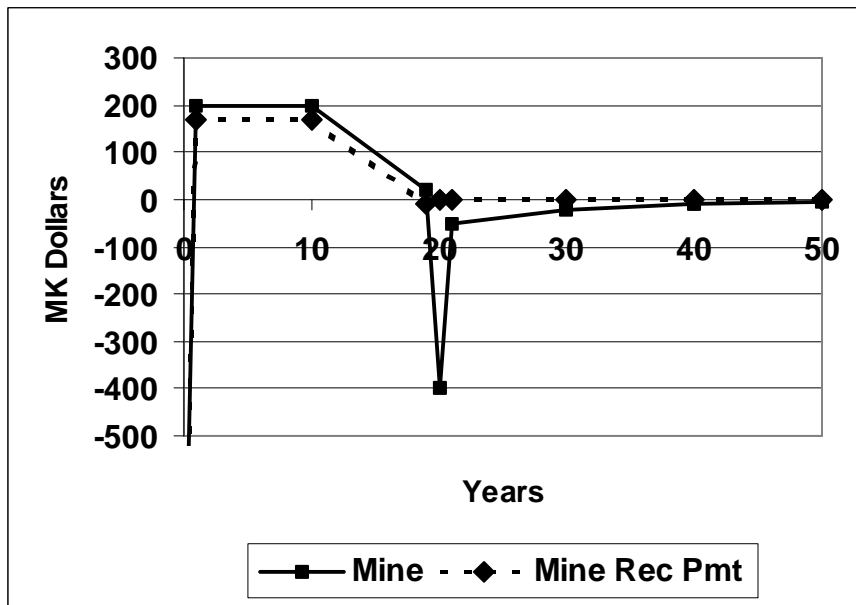


Figure 3. Cash Flow Profile of Mine with Reclamation Payment

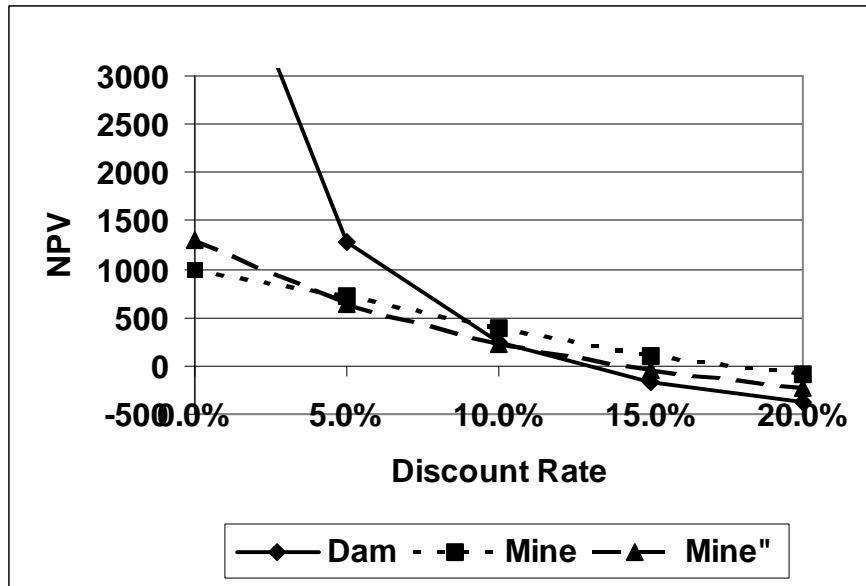


Figure 4. Sensitivity Analysis of NPV and Discount Rates of Mine Options and Dam

Intergenerational Equity

“Now,” continued the Prince, “I am also concerned with the choice of discount rate since both projects span so many years. Should we use a lower discount rate to account for intergenerational equity, such as described in Portney and Weyant (1999)?”

“Prince,” replied the Wizard, “I have read this book. It contains valuable contributions from Solow, Arrow, Dasgupta, Smith, Lind, and many other very notable economists. One of the major conclusions was that intergenerational discounting at some lower positive discount rate should be used with projects that have lives of 50 or more years. While a lower rate could be used in the present case, the time period is marginally long, and there is considerable disagreement about just how and how much to reduce the discount rate. I recommend that we do not use any discount rate lower than 10% in this case. This means we choose the mine.”

Adjusting OCC to Account for Imperfect Analysis

The Prince considered this explanation and then replied “The discount rate still bothers me. What does King Richard in the adjoining kingdom use as his discount rate? His OCC should be much the same as ours since we both have similar investment opportunities.”

“King Richard,” answered the Wizard uses 20%, which is higher than ours is. He would choose the mine as well.”

“Why is his discount rate higher than ours?” asked the Prince?

There was silence as the Wizard and his workers looked at each other and shrugged unknowingly.

From the corner of the room stepped forward an old floppy eared rabbit who had been around the castle as long as anyone could remember. “Prince, I can answer that. The truth is, you have a better Wizard than King Richard, so there is a greater risk of his projects unexpectantly failing or succeeding. King Richard knows this and has added an additional risk factor to his discount rate in the hope of avoiding unexpected losses and thereby increasing his overall rate of return. While this adjustment process has wide appeal, the truth is that it is invalid in this case. Such an adjustment would be valid only if the King’s Wizard consistently under or over estimated the NPV. However, his Wizard’s errors are random.”

“As a result,” continued the rabbit, “increasing the King’s discount rate means more projects are rejected and more capital is left for the default safe investment. If the King raises the discount rate sufficiently high, all projects will be rejected except the default safe investment. This means his resulting OCC becomes the safe rate. You and your Wizard do much better. You should not consider using a higher rate just because King Richard does to value similar projects.”

“Thank you, Rabbit.” exclaimed the Prince. “I now have a considerable amount of information I need to make a choice. I must leave now to meet the magical frog at the pond.”

The Decision

The Prince hurriedly left the castle to seek the frog, who was easily found sitting on the same lily pad as the day before.”

“Hi, Prince!” greeted the frog. “Have you made a decision?”

“I have!” replied the Prince. “I am going to choose the dam.”

“That is a good project, Prince. But, why did you choose the alternative with the lower NPV?” asked the frog.

The Prince replied “The NPV of the mine is higher even with a sinking fund to pay for future pollution control. If all works as planned, the mine would be the correct choice. However, if something happens to me, there may be a temptation for the next ruler to use the accumulated sinking fund for other purposes before the pollution is abated. Alternatively, pollution abatement may be more difficult than expected in spite of what my Royal Engineers have to say.”

“I simply do not want to take the chance of saddling the next generation with a non-productive payment that this current generation has caused, especially since an alternative investment is available. Therefore, I choose the dam.” explained the Prince.

“Congratulations!” exclaimed the frog. “You have considered both the normative economics, which favor the mine, and your own subjective judgement. The application of both normative and subjective economics is important in almost all important investment decisions. Your Wizard supplied the numbers that describe the costs, benefits, and risks of both projects. You then used your subjective reasoning to help interpret what the numbers mean to you and your subjects. This is the proper role of a decision maker. You did well. The dam will now be constructed.”

“Thank you for the project as well as the compliments, frog.” replied the Prince. “But, I worry that if I had additional information, I might have made a better choice.”

“Well, you might have considered option pricing.” replied the frog. “Gotta go now. The Missus is waiting.” And with that, the frog plunged into the pond leaving the Prince scratching his head.

“Option pricing. What’s that?” the Prince asked himself as he turned and started walking back to the castle. “Hmmm,” mused the Prince. “I wonder where the Wizard is?”

Thus ends this fairy tale, as all fairy tails must. And they lived happily ever after.

References

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